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Effects of the motion of dust particles on turbulence transport equations

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A generalized eddy-viscosity function ν_T, is introduced in order to express the Reynolds stress in an incompressible dusty gas as a linear combination of the Kronecker and rate-of-strain tensors. On the basis of Saffman's dusty-gas model a transport equation for the eddy viscosity is derived from the general turbulence energy equations, thereby introducing two additional functions, the specific turbulence kinetic energy E_1, and a scale variable s. In order to determine the three variables modified Prandtl–Wieghardt relation among them is accepted and a transport equation for s is postulated in the same manner as in the clean-gas turbulence transport model (firstly proposed by Harlow & Nakayama 1967) but with the inclusion of an additional term accounting for the dust particles stabilizing action. We are considering values of loading (mass ratio of particles) of order of unity, with particle/gas density ratios of order of 10^3 and volume concentrations of the order of 10^{-3}, so that particle–particle interactions are neglected. Supposing that the particles nearly follow the gas motion, following well at large scales and poorly at small, an application of the theory to problem of numerical calculations of the dusty-gas parameters such as mean velocity profile of turbulent pipe flow is given.

1. Introduction

The observation that adding dust to air flowing in turbulent motion through a pipe can reduce the viscosity by as much as 40% compared with the clean gas, has been reported by Sproull (1961). Saffman (1962) proposed a model of this phenomena and a system of equations describing the motion of a gas carrying a little amount of small particles. He investigated the behaviour of infinitesimal disturbances of a steady laminar flow and he came to some preliminary conclusions concerning the stabilizing action of dust particles. In 1967 Harlow & Nakayama developed a very successful analytical theory of the one-phase incompressible turbulence transport. The success of their theory is due to the excellent mathematical and physical treatment of the classic hydrodynamical closure problem with a minimal introduction of empirical relationships. Our principal suggestion is that the approach by Harlow & Nakayama (1967), which proved to be a satisfactory characterization of the turbulence motion of a clean gas, could be applied to the problem of dusty gas motion as well. In the present paper we use their method to find more general transport equations for the case of dusty gas flow having as a starting point Saffman's dusty gas model. Finally, an application of the general theory to the problem of the influence of turbulence characteristics on the mean velocity profile of turbulent pipe flow is given. The dusty pipe flow equations are solved numerically and the results are compared with the known solution for one-phase pipe flow.

2. The energy transport equations

Let us assume that Saffman's dusty-gas model is completely acceptable as a description of the incompressible gas carrying small solid particles. Accordingly, the derivations are based on the following set of equations:

$$\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_j^2}-\frac{KN}{\rho}(u_i-v_i), \tag{2.1}$$

$$\frac{\partial u_i}{\partial x_i} = 0, \tag{2.2}$$

$$\frac{\partial v_i}{\partial t}+v_j\frac{\partial v_i}{\partial x_j} = -\frac{K}{m_s}(v_i-u_i). \tag{2.3}$$

In equations (2.1)–(2.3),

$u_i(x_j, t)$ are the gas velocity components;
$v_i(x_j, t)$ are the solid particles (dust) velocity components;
$p(x_j, t)$ is the unknown pressure field;
$\rho =$ const. is the gaseous mass density per unit volume;
$m_s = \frac{4}{3}\pi R_s^3\rho_s$ is the mass of a single spherical dust particle of radius R_s;
$\rho_s =$ const. is the density of the material in the dust particles;
$\nu =$ const. is the molecular kinematic viscosity;
$K = 6\pi R_s\rho\nu$ by the Stokes drag formula;
$N =$ const. is the number density of dust particles.

It is supposed that the dust particles are uniform in size and shape and that their mass concentration (loading) $\mathscr{L} \equiv m_s N/\rho$ is of order unity (for common materials $\rho s/\rho \sim 10^3$ so that the bulk concentration of particles $\frac{4}{3}\pi R_s^3 N$ is of order 10^{-3}).

The field variables u_i, v_i and p/ρ are split into mean and fluctuating parts:

$$\left.\begin{aligned} u_i &= \bar{u}_i+u_i', \\ v_i &= \bar{v}_i+v_i', \\ p/\rho = P &= \bar{P}+P', \end{aligned}\right\} \tag{2.4}$$

so that, by definition, $\overline{u_i'} = \overline{v_i'} = \overline{P'} = 0$. The equations for the mean flow field are readily obtained from (2.1)–(2.4) in the following form

$$\frac{\partial \bar{u}_i}{\partial t}+\bar{u}_k\frac{\partial \bar{u}_i}{\partial x_k}+\frac{\partial}{\partial x_k}\overline{u_i' u_k'}+\frac{\partial \bar{P}}{\partial x_i} = \nu\frac{\partial^2 \bar{u}_i}{\partial x_k^2}-\frac{KN}{\rho}(\bar{u}_i-\bar{v}_i), \tag{2.5a}$$

$$\frac{\partial \bar{v}_i}{\partial t}+\bar{v}_k\frac{\partial \bar{v}_i}{\partial x_k}+\frac{\partial}{\partial x_k}\overline{v_i' v_k'} = -\frac{K}{m_s}(\bar{v}_i-\bar{u}_i), \tag{2.5b}$$

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0, \tag{2.5c}$$

while subtraction of (2.5*a*) and (2.5*b*) from (2.1) and (2.3) respectively leads to the equations for the fluctuating dusty-flow velocities and pressure fields components:

$$\frac{\partial u_i'}{\partial t}+\frac{\partial}{\partial x_k}\{\bar{u}_k u_i'+\bar{u}_i u_k'+u_i'u_k'-\overline{u_i'u_k'}\}+\frac{\partial P'}{\partial x_i}=\nu\frac{\partial^2 u_i'}{\partial x_k^2}+\frac{KN}{\rho}(v_i'-u_i'), \tag{2.6a}$$

$$\frac{\partial v_i'}{\partial t}+\frac{\partial}{\partial x_k}\{\bar{v}_k v_i'+v_k'\bar{v}_i+v_k'v_i'-\overline{v_k'v_i'}\}=\frac{K}{m_s}(u_i'-v_i'). \tag{2.6b}$$

As it is seen from (2.5*a*) the influence of the turbulence momentum transport upon a mean velocity profile $\bar{u}_i$ is described by the Reynolds stress tensor $\overline{u_i'u_j'}$. Our purpose here is two fold. On one hand, to obtain some information concerning the time–space evolution of this tensor from the relevant transport equation. On other hand, to find the influence of the dust particles on the reduction of turbulence momentum. Multiplication of (2.6*a*) by u_j', averaging and adding the same relation with interchanged indices leads to the result

$$\begin{aligned}\frac{\partial}{\partial t}\overline{u_i'u_j'}+\bar{u}_k\frac{\partial}{\partial x_k}\overline{u_i'u_j'}+\overline{u_i'u_k'}\frac{\partial \bar{u}_j}{\partial x_k}+\overline{u_j'u_k'}\frac{\partial \bar{u}_i}{\partial x_k}\\ +\frac{\partial}{\partial x_k}\overline{u_i'u_j'u_k'}+\overline{u_i'\frac{\partial P'}{\partial x_j}}+\overline{u_j'\frac{\partial P'}{\partial x_i}}=\nu\left\{\overline{u_j'\frac{\partial^2 u_i'}{\partial x_k^2}}+\overline{u_i'\frac{\partial^2 u_j'}{\partial x_k^2}}\right\}\\ +2\frac{K}{m_s}\mathscr{L}\left\{\frac{\overline{u_i'v_j'}+\overline{v_i'u_j'}}{2}-\overline{u_i'u_j'}\right\},\end{aligned} \tag{2.7}$$

where $\mathscr{L}\equiv m_s N/\rho$.

In the same manner an equation describing the time–space evolution of the tensor $\frac{1}{2}(\overline{u_i'v_j'}+\overline{u_j'v_j'})$ may be deduced from (2.6*a*) and (2.6*b*). It is obvious that the mutual gas–particle correlator (which appears in (2.7)) depends on the $\overline{u_i'u_j'}$ correlator and on higher-order terms of the correlation as well. Let us assume that the solid particles follow fairly well the mean (large-scale) motion

$$\bar{u}_i=\bar{v}_i, \tag{2.8}$$

although they do not follow the high-frequency stochastic oscillations of the gas flow, i.e.

$$\tfrac{1}{2}(\overline{u_i'v_j'}+\overline{v_i'u_j'})\ll\overline{u_i'u_j'}. \tag{2.9}$$

We consider further the 'relaxation time' of the dust particles m_s/K (the time which they need to adjust to changes in the gas velocity) as being much greater than the time scale of the most unstable disturbance,

$$m_s/K\gg L/u_m \tag{2.9a}$$

(here L denotes the length scale and u_m is the velocity scale of the mean flow field). In these circumstances the relation (2.9) will be satisfied.

In reality there are various physical mechanisms controlling the interaction between the solid particles and the driving gas flow (see, for example, Owen 1969). But from a mathematical point of view it is obvious that after the assumptions (2.8) and (2.9) the dust-particles motion equations (2.5*b*) and (2.6*b*) should not be used in the formal derivations. Thus the only equation which should be considered in order to describe the turbulence transport of the dusty-gas flow is equation (2.7).

The basic assumption concerning the Reynolds stress itself is the same as firstly proposed by Harlow & Nakayama (1967):

$$u_i'u_j' = \tfrac{2}{3} E_1\delta_{ij} - \nu_T e_{ij}, \tag{2.10}$$

where e_{ij} is the symmetric rate-of-strain tensor

$$e_{ij} \equiv \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}, \tag{2.11}$$

ν_T is the kinematic eddy-viscosity coefficient,

$$E_1 \equiv \tfrac{1}{2}(u_i')^2 \tag{2.12}$$

is the local instantaneous specific kinetic energy of turbulent fluctuations. Thus the problem of obtaining an equation for the mean dusty-gas dynamics reduces to that of finding $\bar{E}_1$ and ν_T as functions of position and time. From (2.7) after contracting the indices and inserting (2.10) we get

$$\frac{\partial \bar{E}_1}{\partial t} + \bar{u}_k \frac{\partial \bar{E}_1}{\partial x_k} - \nu_T \bar{e}_{ik} \frac{\partial \bar{u}_i}{\partial x_k} + \frac{\partial}{\partial x_k} \overline{E_1' u_k'} + \frac{\partial}{\partial x_k} \overline{P' u_k'} = \nu \left\{ \frac{\partial^2 \bar{E}_1}{\partial x_k^2} - \overline{\left(\frac{\partial u_i'}{\partial x_k}\right)^2} \right\} - \frac{\bar{E}_1}{\tau}, \tag{2.13}$$

where

$$\tau \equiv \frac{\rho}{2KN} = \frac{m_s/K}{2\mathscr{L}} \tag{2.14}$$

is the characteristic time scale for the rate of mean turbulence energy attenuation due to gas-element–solid-particles interaction. By analogy with the Harlow & Nakayama (1967) consideration of the rate of dissipation of turbulence energy due to molecular viscosity ν, we write

$$\nu \overline{\left(\frac{\partial u_j'}{\partial x_k}\right)^2} = \frac{2\Delta\nu}{s^2} \bar{E}_1, \tag{2.15}$$

where the dimensionless function Δ is written in the form

$$\Delta = \beta(1 + \delta\nu_T/\nu), \quad \beta = \text{const.}, \quad \delta = \text{const.}, \tag{2.16}$$

and the variable s is introduced as a measure of the eddy-size scale appropriate to the dissipative process under consideration. It is convenient to introduce a slightly modified Prandtl–Wieghardt relation among the three variables $\bar{E}_1$, ν_T and s

$$\bar{E}_1 = \frac{1}{2\gamma}\left(\frac{\nu_T}{s}\right)^2 \tag{2.17}$$

where the constant γ is expected to be near unity. Continuing the development from (2.13) we represent the turbulent diffusion of the two scalar quantities E_1' and P' by the following flux approximations

$$\overline{u_k' E_1'} = -\alpha\nu_T \frac{\partial \bar{E}_1}{\partial x_k}, \tag{2.18}$$

$$\overline{u_k' P'} = -\frac{\theta}{\gamma}\nu_T \frac{\partial \bar{P}}{\partial x_k}, \tag{2.19}$$

in which α, θ are universal dimensionless constants of order unity in magnitude and γ is introduced in accordance with (2.17).

After all these developments (2.13) becomes

$$\frac{\partial}{\partial t}\left(\frac{\nu_T}{s}\right)^2+\bar{u}_k\frac{\partial}{\partial x_k}\left(\frac{\nu_T}{s}\right)^2 = 2\gamma\nu_T\bar{e}_{ik}\frac{\partial\bar{u}_i}{\partial x_k}+\frac{\partial}{\partial x_k}\left[(\nu+\alpha\nu_T)\frac{\partial}{\partial x_k}\left(\frac{\nu_T}{s}\right)^2\right]+2\theta\frac{\partial}{\partial x_k}\left[\nu_T\frac{\partial\bar{P}}{\partial x_k}\right]-\frac{2\nu\Delta\nu_T^2}{s^4}-\frac{\nu_T^2}{\tau s^2}. \tag{2.20}$$

Combining equations (2.8), (2.10) and (2.5*a*) and omitting bars for the dynamics of mean flow of dusty gas we get

$$\frac{\partial u_i}{\partial t}+u_k\frac{\partial u_i}{\partial x_k}=-\frac{\partial}{\partial x_i}\left\{P+\frac{\nu_T^2}{3\gamma s^2}\right\}+\frac{\partial}{\partial x_k}\left\{(\nu+\nu_T)\frac{\partial u_i}{\partial x_k}\right\}+\frac{\partial\nu_T}{\partial x_k}\frac{\partial u_k}{\partial x_i}. \tag{2.21}$$

On the basis of simple physical considerations Harlow & Nakayama (1967) suggested the eddy-size scale transport equation to be constructed by equalizing the sum of terms representing the mean flow convective transport, the viscous and turbulence diffusive transports to the sum of the appropriate source terms. The latter are written in analytical form and contain some dimensionless universal constants. The numerical values of these constants are chosen in such a way that the best fitting of the analytical theory and the known experimental data (Batchelor & Townsend 1948; Reichardt 1951) to be achieved. Without reproducing their theoretical reasoning, just including by an analogy an additional source term

$$\left(\frac{\partial s}{\partial t}\right)_{\text{dusty part.}}=-C\frac{s}{\tau}, \tag{2.22}$$

with C as a universal dimensionless constant and τ defined by (2.14), we postulate the following scale transport equation for dusty-gas flow:

$$\frac{\partial s}{\partial t}+u_k\frac{\partial s}{\partial x_k}-\frac{\partial}{\partial x_k}\left[(\nu+\psi\nu_T)\frac{\partial s}{\partial x_k}\right] = -\frac{2s}{3\nu_T}\left(\frac{\partial\nu_T}{\partial t}+u_k\frac{\partial\nu_T}{\partial x_k}\right)+\frac{\nu_T}{s}F\left(\frac{\nu_T}{\nu}\right)-\left(\frac{\partial u_j}{\partial x_k}\right)^2\frac{s^3}{\nu_T}g\left(\frac{\nu_T}{\nu}\right)-\frac{Cs}{\tau}. \tag{2.23}$$

Here the functions $F(\xi)$ and $g(\xi)$ ($\xi=\nu_T/\nu$), as given in the paper of Harlow & Nakayama (1967), are equal to

$$F(\xi)=\tfrac{1}{3}f_1(\xi)+f_2(\xi), \tag{2.24}$$

$$g(\xi)=g\left(\frac{\xi}{\xi+\xi_2}\right)^m, \tag{2.25}$$

with $f_1(\xi)$ and $f_2(\xi)$ are defined from

$$\xi f_1(\xi)=\mathscr{K}_1(1+\tanh[\tfrac{3}{2}(\xi-\xi_1)]), \tag{2.26}$$

$$f_2(\xi)=\mathscr{K}_2\left(\frac{\xi/\xi_3}{\xi/\xi_3+1}\right)^n, \tag{2.27}$$

$\mathscr{K}_1$, $\mathscr{K}_2$, ξ_1, ξ_2, ξ_3, m, n being constants. Their typical values will be discussed further.

3. Turbulent pipe flow of dusty gas

As a trial application of the general theory given in previous section let us consider a fully developed steady turbulent pipe flow of dusty gas with both gas and solid particles mean velocities having only longitudinal components. With respect to a cylindrical co-ordinate system (r, ϕ, ζ) with ζ measured along the pipe axis it means

$$\mathbf{v} = \mathbf{u} = (0, 0, u(r)). \tag{3.1}$$

For fully developed pipe flow the kinematic eddy-viscosity coefficient ν_T and the scale variable s, being functions of r co-ordinate only, satisfy equations (2.20) and (2.23) in the following simplified form:

$$-\frac{1}{r}\frac{d}{dr}\left\{r(\nu+\alpha\nu_T)\frac{d}{dr}\left(\frac{\nu_T}{s}\right)^2\right\}-\frac{2\theta}{r}\frac{d}{dr}\left\{r\nu_T\frac{\partial P}{dr}\right\}=2\gamma\nu_T\left(\frac{du}{dr}\right)^2-2\beta(\nu+\delta\nu_T)\frac{\nu_T^2}{s^4}-\frac{\nu_T^2}{\tau s^2}, \tag{3.2}$$

$$-\frac{1}{r}\frac{d}{dr}\left\{r(\nu+\psi\nu_T)\frac{ds}{dr}\right\}=\frac{\nu_T}{s}F\left(\frac{\nu_T}{\nu}\right)-\left(\frac{du}{dr}\right)^2\frac{s^3}{\nu_T}g\left(\frac{\nu_T}{\nu}\right)-C\frac{s}{\tau}. \tag{3.3}$$

In these equations r is the distance from the axis of the pipe, the wall being at $r = R$. With λ denoting the constant pressure gradient, the r and ζ components of equation (2.21) become

$$\frac{\partial}{\partial r}\left[P+\frac{\nu_T^2}{3\gamma s^2}\right]=0, \tag{3.4a}$$

$$\frac{du}{dr}=-\frac{\lambda r}{2(\nu+\nu_T)}. \tag{3.4b}$$

The equations (3.2) and (3.3) can be combined with equations (3.4a, b) and written in the following dimensionless form

$$-\frac{1}{x}\frac{d}{dx}\left\{x(1+\alpha_1\xi)\frac{d}{dx}\left(\frac{\xi}{z}\right)^2\right\}=\frac{\gamma x^2}{2\kappa^4}\frac{\xi}{(1+\xi)^2}-\frac{2\beta\xi^2(1+\delta\xi)}{z^4}-\mathscr{D}\frac{\xi^2}{z^2}, \tag{3.5}$$

$$-\frac{1}{x}\frac{d}{dx}\left\{x(1+\psi\xi)\frac{dz}{dx}\right\}=\frac{\xi}{z}\{\tfrac{1}{3}f_1(\xi)+f_2(\xi)\}-\frac{x^2z^3}{4\kappa^4}\frac{g(\xi)}{(1+\xi)^2}-C\mathscr{D}z, \tag{3.6}$$

where

$$\alpha_1=\alpha-\frac{2\theta}{3\gamma},\quad r=Rx,\quad \xi=\frac{\nu_T}{\nu},\quad z=\frac{s}{R};\quad \kappa=\nu\lambda^{-\frac{1}{2}}R^{-\frac{3}{2}},\quad \mathscr{D}=12\pi R^2R_sN. \tag{3.7}$$

From (3.4b) and (3.7) it is clear that the dimensionless parameter κ is related to the Reynolds number $Re_L = Ru_{\max}/\nu$ for laminar conditions ($\xi \equiv 0$) by means of $(Re_L)^{-\frac{1}{2}} = 2\kappa$. The coupled ordinary nonlinear differential equations (3.5), (3.6) are to be solved subject to the boundary conditions

$$\left.\begin{aligned} d\xi/dx = dz/dx = 0 \quad &\text{at} \quad x = 0,\\ \xi = z = 0 \quad &\text{at} \quad x = 1.\end{aligned}\right\} \tag{3.8}$$

The source functions $f_1(\xi)$, $f_2(\xi)$ and $g(\xi)$ which appear in equation (3.6) were defined by relations (2.24)–(2.27). The solution of equations (3.5) and (3.6) was found in numerical calculations by use of Runge–Kutta method. The most values of dimensionless constants introduced in previous section was taken into consideration following Harlow & Nakayama (1967), which have reported satisfactory agreement of their numerical results and experimental data. Namely,

$$\alpha_1 = 1, \quad \gamma = 2, \quad \beta = 5, \quad \delta = 10^{-2}, \quad \psi = 1, \quad \kappa = 4{\cdot}83\times 10^{-5},$$
$$g = 4\times 10^{-2}, \quad \xi_1 = 7, \quad \xi_2 = \xi_3 = 100, \quad m = n = 2,$$
$$\mathscr{K}_1 = 7{\cdot}5, \quad \mathscr{K}_2 = 0{\cdot}002 \text{ or } 0{\cdot}0005$$

in addition to

$$0 \leqslant \mathscr{D} \leqslant 10^3, \quad 0{\cdot}1 \leqslant C \leqslant 10 \tag{3.9}$$

were used.

It is worth to remind here, that the $\mathscr{D}$ value in the present model is accounting for the influence of solid particles on turbulence transport in a bounded gas flow: in particular, the results of Harlow and Nakayama 1967 refer to $\mathscr{D} = 0$, but in the experiments of Sproull 1961 $10^2 \leqslant \mathscr{D} \leqslant 5\times 10^3$. It should be noted that the conditions (3.8) give us the quantities of the functions z, ξ and their derivatives in respect of x in different ends of the interval (0, 1). For this reason the integration started at $x = 0$ for a set of $\xi_0 = \xi(x=0)$ and $z_0 = z(x=0)$ was repeated for other ξ_0, z_0 so that by variation of ξ_0, z_0 to find the appropriate set, resulting in $z(1) = \xi(1) = 0$. In fact this variation stopped if $z(x)_{\text{final}} < 0{\cdot}05 z_0$, $\xi(x_{\text{final}}) < 0{\cdot}05 z_0$ for $1 - x_{\text{final}} \approx 10^{-4}$, because at $x \to 1$ the derivative $\xi'(x)$ becomes extremely large and small variation of ξ_0, z_0 lead to the substantial changes in final ξ, z.

The numerical calculations were performed for six values of the dusty parameter $\mathscr{D}(0; 0{\cdot}1; 1; 10; 10^2; 10^3)$ and three values of universal constant C (0·1; 1; 10). Variations of ξ and z as a function of x are shown on figure 1 for two different $\mathscr{D}$ values (1 and 10^3). The form of $\xi(x)$ is the same as that curve derived by Harlow & Nakayama (1967), but the addition of dust (the increase of $\mathscr{D}$ from 1 to 10^3 at $\mathscr{K}_2 = 0{\cdot}002$ and $C = 1$) leads to decrease of eddy viscosity by as much as 20 % in the core of turbulent dusty flow ($x \approx 0$) and less than 1 % in the boundary layer ($x \approx 1$). The dependence of the kinematic eddy viscosity coefficients $\xi_0(\xi(x)$ at $x = 0)$ upon the $\mathscr{D}$ parameter is demonstrated on figure 2 for several values of constant C and two values of $\mathscr{K}_2$. The same dependence for the eddy size scale z_0 ($z(x)$ at $x = 0$) is shown on figure 3. It is seen from these figures that the numerical experiment with reduction of $\mathscr{K}_2$ from 0·002 to 0·0005 results in substantial reduction of ξ_0 and z_0 in the whole interval $0 \leqslant \mathscr{D} \leqslant 10^3$ and thus the value $\mathscr{K}_2' = 0{\cdot}0005$ does not square with Harlow & Nakayama description of turbulence transport. It is important to note that appreciable reduction of eddy viscosity coefficient and size scale is achieved only if $\mathscr{D}$ is sufficiently high ($\mathscr{D} \gtrsim 10^2$). This conclusion is confirmed also by figure 4, which illustrates the increase of the mean velocity at $x = 0$ as a function of the $\mathscr{D}$ parameter in qualitative agreement with Sproull's (1961) observations.

4. Discussion

The picture of the turbulence transport in an incompressible dusty-gas flow emerging from the conceptual model proposed is the following: A turbulent motion which changes in time much more rapidly than the mean fluid motion extracts energy from the mean shear flow and the turbulent transport of several mechanical quantities may be represented in appropriate flux approximations. It is supposed that for solid

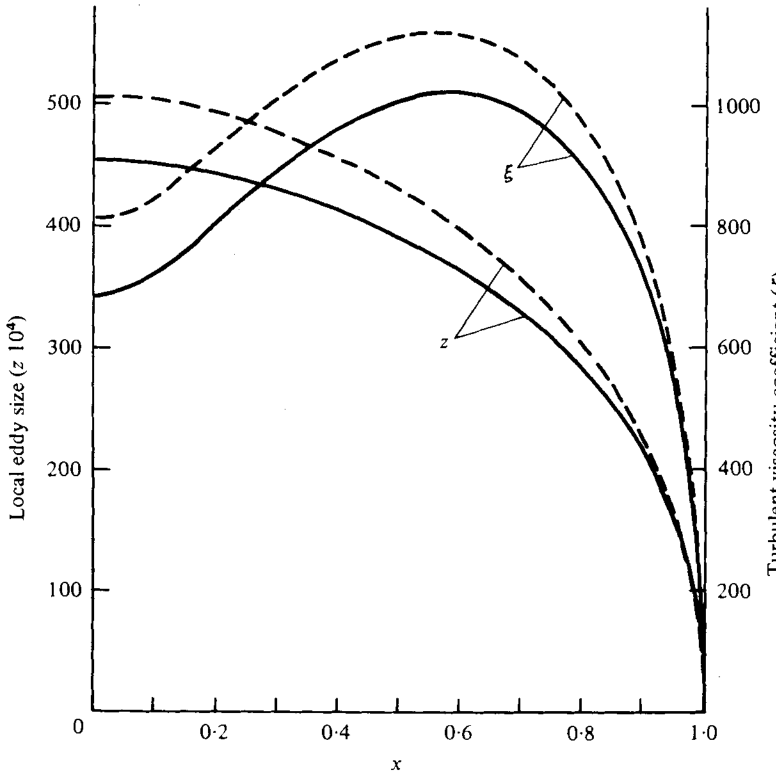

FIGURE 1. Variation of ξ and z as a function of radius. $C = 1$, $K_2 = 0{\cdot}002$; - - - $D = 1$; ——, $D = 1000$.

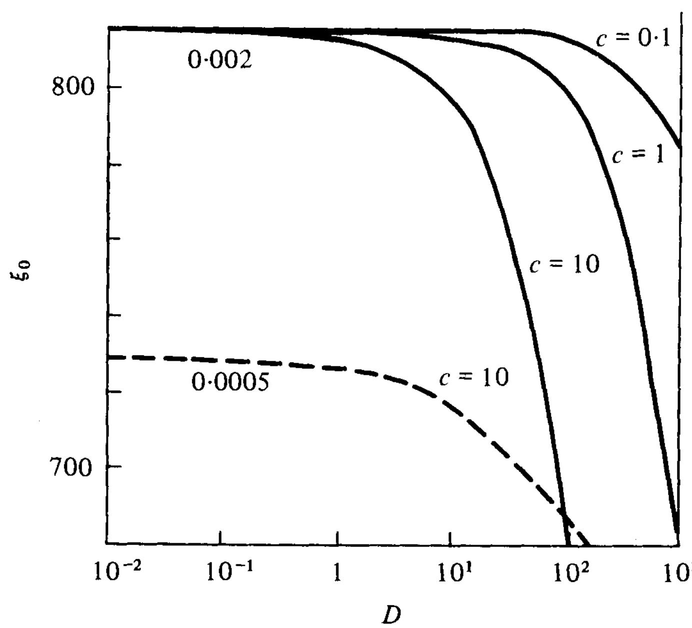

FIGURE 2. The dependence of the kinematic eddy viscosity coefficient $\xi_0(\xi(x)$ at $x = 0)$ on the parameter D. - - -, $K_2 = 0{\cdot}0005$, $C = 10$; ——, $K_2 = 0{\cdot}002$, $C = 0{\cdot}1$, 1, 10.

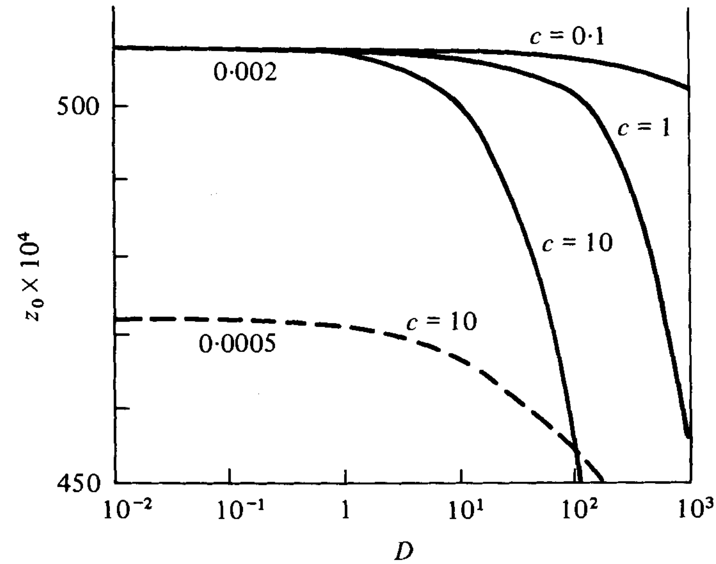

FIGURE 3. The normalized eddy size scale $z_0 = s_0/R$ versus the D parameter ($z_0 = z(x)$ at $x = 0$). - - -, $K_2 = 0{\cdot}0005$, $C = 10$; ——, $K_2 = 0{\cdot}002$, $C = 0{\cdot}1, 1, 10$.

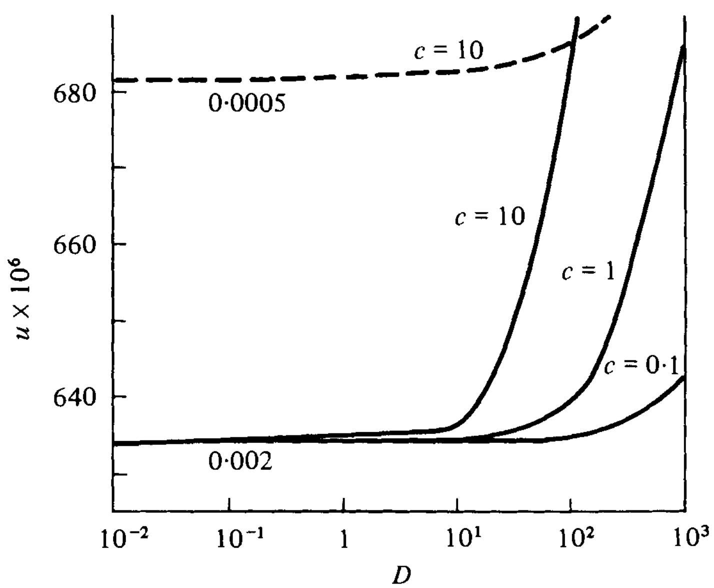

FIGURE 4. Mean velocity $u = \bar{u}(x = 0, D \neq 0)$ as a function of the D parameter.

particles uniformly distributed in the gas volume the absolute value of the mutual correlator of the gas element and the solid particles fluctuational velocities is much less than the Reynolds stress tensor for every pair of indices i, j and at all space-time points. The result is a pair of transport equations for the turbulence energy and scale, together with a relationship among these variables and the eddy kinematic viscosity coefficient. On the basis of the model proposed the phenomenon of turbulent drag reduction due to a small amount of solid particles appears most easily explained as being caused by: firstly, the dissipation of turbulence energy due to the time lag between acceleration of a gas element and its inertial reaction and secondly, the additional breakdown of eddy size in the presence of dust.

The influence of the modified by solid particles turbulence characteristics upon the mean velocity profile was investigated by solving the turbulence transport equations restricted to dusty pipe flow. The numerical study of these equations shows that an appropriate choice of the universal constants leads to the reduction of the viscosity of dusty gas by as much as 20 % compared with the clean gas.

Further development of the analitical theory of turbulence in dusty gas flows needs more detail investigation of the governing system of equations (2.1)–(2.3) and new experimental investigations allowing direct comparison with this and future theoretical models.

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